Quantum Field Theory

Syllabus for quarter 1

The page numbers mentioned in this short course overview refer to the reader of the course. More extended information can be found on Brightspace in the weekly content sections.

Lecture 1: page 1-6, Ex.1 + part of Ex.2

- Relativistic theories: fields or particles? $\Leftrightarrow$ local laws of physics, Compton wavelength.
- Lagrangian formalism for continuous fields, relativistic equations of motion from Hamilton's variation principle (Euler-Lagrange equations), Hamiltonian density.

Lecture 2: page 6-13, part of Ex.2 + Ex.3

- Relativity principle $\Leftrightarrow$ scalar Lagrangian densities.
- Noether's theorem for continuous symmetries (conserved currents & charges).
- Examples in lecture 2: translation symmetry (conserved energy-momentum tensor, energy and momentum) and global U(1) gauge symmetries (conserved particle number).
  Reading material: Lorentz symmetry (conserved tensor and angular momentum).
- Canonical quantization of field theories in the Schrödinger picture (using a t=0 snapshot).

Lecture 3: page 13-19, Ex.4

- Quantization of free field theories in terms of linear harmonic oscillators (creation and annihilation operators, particles = fundamental energy quanta, comparison with the canonical approach).
- Example: free real Klein-Gordon theory (energy spectrum bounded from below, zero-point energy, fundamental energy quanta are spin-0 bosons = antibosons).
- Lorentz-invariant normalization of states and integration measure, particles as extended objects.

Lectures 4a+b: page 19-27 and 28-33, two lectures in week 4 and no exercise class

- Switching on time (Schrödinger picture $\rightarrow$ Heisenberg picture): causality, propagation amplitudes, why t=0 quantization suffices.
- Complex Klein-Gordon theory (see Ex.4): causality, propagation amplitudes, notion of antiparticles and particle number.
- Inversion of the Klein-Gordon operator: Green's function method and Feynman propagator (= time-ordered propagation amplitude).
- Interacting scalar theories (non-conserved # particles, change in particle interpretation).
- Viability of perturbation theory: dimensional analysis $\Leftrightarrow$ smallness of interactions, issues with high-energy (UV) divergences, renormalizable field theories vs non-renormalizable effective field theories (EFT's).
- Example: Fermi-model of weak interactions.
- Perturbation theory kick-off for Φ^4-theory and the scalar Yukawa theory.

Lecture 5: page 33-40, Ex.5+6

- Setting up perturbation theory: rewriting Heisenberg-picture objects in interaction-picture (free-theory) form.
- Wick's theorem for bringing time-ordered products of fields in normal-ordered form: the notion of contractions + the link to Feynman propagators.
- Green's functions: Heisenberg picture vs interaction picture (Riemann-Lebesgue lemma, time-evolution S-operator).

Lecture 6: page 41-51, Ex.7+8 [Φ^4-theory in the lecture, scalar Yukawa theory in the exercises]

- Diagrammatic notation of perturbation theory for Green's functions: Feynman diagrams and rules in position space and momentum space (propagators, external points, vertices = internal points, energy-momentum conservation at each vertex, symmetry factors).
- Diagrammatic notation for scattering amplitudes (intial/final state, S- and T-matrix, matrix element, arrow of time, external lines, fully connected diagrams, loop corrections, amputation procedure, arrow convention).

Lecture 7: page 51-64, Ex.9 + part of Ex.11

- Non-relativistic limit of elastic scattering in Φ^4-theory and scalar Yukawa theory: forces between particles (contact interactions, force mediators, attractive vs repulsive forces + the link to the spin of the force mediator).
- Translating matrix elements into probabilities for decay and scattering: Lorentz-invariant final-state phase-space element, final-state identical particle factors, decay rates, (partial) decay width, scattering rates, (differential) cross sections, centre-of-mass kinematics, reaction channels, 2-body phase-space element.

Exercises:

- **1: Classical example of going from a discrete to a continuous system**
- **2: Free electromagnetic theory (dealing with multiple Minkowski indices)**
- **3: Example of extended gauge symmetries**
- **4: The complex Klein-Gordon theory (particle interpretation & vacuum properties)**
- **5: Time-ordered propagation amplitudes for the complex Klein-Gordon theory**
- **6: Time-ordered exponentials (which feature in perturbation theory)**
- **7: Feynman rules for the scalar Yukawa theory and arrow convention**
- **8: More on the arrow convention for particles and antiparticles**
- **9: Lowest-order amplitudes for various processes in the scalar Yukawa theory**
- **11: Decay rates, cross sections and reaction channels for scalar-Yukawa-theory processes**