Standard Model and Beyond Exercises week 8

Exercise 13: And YOU get mass! And YOU get mass!

Consider the gauged kinetic Higgs Lagrangian term $(D_\mu\Phi)^\dagger(D^\mu\Phi)$, with

$$D_\mu\Phi = (\partial_\mu + \frac{i}{2}g\vec{\tau}\cdot\vec{W}_\mu + \frac{i}{2}g'B_\mu)\Phi \tag{1}$$

where g is the $SU(2)_L$ gauge coupling, g' the $U(1)_Y$ gauge coupling, $W_\mu^{1,2,3}$ the $SU(2)_L$ gauge bosons, $\tau^{1,2,3}$ the Pauli spin matrices and B_μ the $U(1)_Y$ gauge boson.

(a) Insert the vacuum expectation value $\langle\Phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$ in this lagrangian and use that $(\vec{\tau}\cdot\vec{f})^2 = \vec{f}^{\,2}I$ for arbitrary functions f^1, f^2 and f^3, with I being the 2×2 unit matrix. Derive in this way the following expression for the gauge-boson mass terms:

$$\begin{aligned}\mathcal{L}_{\text{mass}}^{\text{GB}} = \frac{1}{2}\Big\{&\frac{1}{4}v^2g^2\left(W_\mu^1W^{1,\mu} + W_\mu^2W^{2,\mu}\right) \\ &+ \frac{1}{4}v^2\left(g^2W_\mu^3W^{3,\mu} + g'^2B_\mu B^\mu - gg'\left[W_\mu^3B^\mu + B_\mu W^{3,\mu}\right]\right)\Big\}\end{aligned} \tag{2}$$

(b) The diagonalized gauge-boson mass terms read $+\frac{1}{2}\sum_{a=1}^4 M_a^2V_\mu^aV^{a,\mu}$. Find the four eigenvalues M_a^2 and express the corresponding eigenvectors V_μ^a in terms of the original fields $W_\mu^{1,2,3}$ and B_μ.

Hint: use matrix notation for $\mathcal{L}_{\text{mass}}^{\text{GB}}$.

(c) What do the eigenvalues and eigenvectors you found in (b) represent?

(d) What would change if we would have used instead of Φ a scalar doublet $\tilde{\Phi}$ with opposite hypercharge, such that $\langle\tilde{\Phi}\rangle_0 = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$ and $D_\mu\tilde{\Phi} = \left(\partial_\mu + \frac{i}{2}g\vec{\tau}\cdot\vec{W}_\mu - \frac{i}{2}g'B_\mu\right)\tilde{\Phi}$?

In (c) you found two mixed states of W_μ^3 and B_μ. If you write these fields in a vector (W_μ^3, B_μ), you can directly find the mixed states by applying a rotation matrix

$$\begin{bmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{bmatrix} \equiv \begin{bmatrix} c_w & s_w \\ -s_w & c_w \end{bmatrix} \tag{3}$$

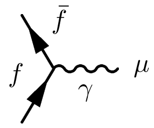
to it. This leaves θ_w as a free parameter. It is called the electroweak mixing angle or Weinberg angle..

(e) Give an expression for $\tan\theta_w$ in terms of g and g'.

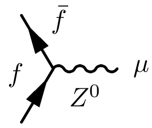
It is striking that the seemingly random sizes of the particle masses (0 GeV, 80.3 GeV and 91.2 GeV for the photon, W boson and Z boson respectively) have a common origin. The theoretically beautiful notion of unification of forces has inspired many to look for a theory that also includes the strong force, creating a so called *unified gauge theory*, or *grand unified theory* (GUT). Although the combination of the EM and weak force in the exercise above is a nice concept, it in itself is not a unified theory, because the coupling strengths g and g' are independent of each other. Unfortunately for the GUT enthusiasts a true grand unified theory has not yet been found to be realised in nature. Additionally: the theoretical power of the theory stops at exactly the point you finished exercise (e): the rotation angle you found there is one of those parameters that you have to measure to know. There is – as far as we know – no way to derive its value from the standard model theory. In the exercise of last week you did half of this measurement (the mass of the Z boson) yourself already!

Exercise 14: One theory to (Feynman) rule EM and weak interactions

(a) Write down the $SU(2)_L \times U(1)_Y$ gauge interaction term for neutral current (NC) interactions (interactions involving a neural gauge boson: γ or Z^0) and derive the following Feynman rules for the indicated vertices (all particles are defined to be incoming):



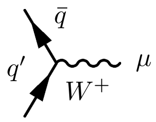
$$-i|e|Q_f\gamma^\mu$$



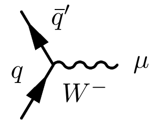
$$-\frac{ig}{c_w}\gamma^\mu(c_v^f - c_A^f\gamma^5)$$

where $c_v = \frac{1}{2}I_3(f) - s_w^2 Q_f$ is the vector coupling of the Z^0 boson, $c_A = \frac{1}{2}I_3(f)$ its axial vector coupling and s_w the sine of the Weinberg angle.

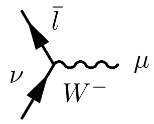
(b) Do the same for charged current (CC) interactions (interactions involving a charged gauge boson W^+ or W^-), where q denotes an up-type quark and q' a down-type quark.



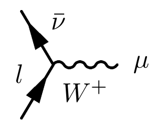
$$-\frac{ig}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$



$$-\frac{ig}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$



$$-\frac{ig}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$



$$-\frac{ig}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$

The *branching ratio* for a proccess $A \to BC$ is the fraction of A particles that decay to a B and C particle.

(c) Assume a W^- boson is produced in a collider experiment and that it can only decay into two particles whose summed mass is smaller than (or equal to) the W mass. We make the simplification that we assume all produced particles to be massless. What is the branching ratio for this boson to $d\bar{u}$? And what it the branching ratio to $e\bar{\nu}_e$? And $c\bar{s}$?

You might have noticed that neutral current interactions leave the particle type/flavour invariant, while charged current interactions can change it. This is experimentally an important notion: finding a *flavour changing neutral current* (FCNC) would be a huge sign of physics beyond the standard model and is therefore actively sought for in particle physics experiments.