Quantum Field Theory Exercises week 6

Exercise 7: Feynman rules for the scalar Yukawa theory + arrow convention

Take the Lagrangian for the scalar Yukawa theory:

$$\mathcal{L} \;=\; (\partial_\mu\psi^*)(\partial^\mu\psi) \;+\; \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) \;-\; M^2\psi^*\psi \;-\; \frac{1}{2}m^2\phi^2 \;-\; g\psi^*\psi\phi \;,$$

with $\phi \in \mathbb{R}$ and $\psi \in \mathbb{C}$.

(a) Which term in the Lagrangian gives rise to an interaction?

(b) Calculate up to first order in g:

$$\langle 0|\, T\Big(\hat{\psi}_I^\dagger(x_1)\hat{\phi}_I(x_2)\hat{\psi}_I(x_3)\, e^{-i\int d^4x\, \hat{\mathcal{H}}_I(x)}\Big)\,|0\rangle \;,$$

indicating the Feynman propagator for the ψ field by

$$\langle 0|\, T\big(\hat{\psi}_I(x)\hat{\psi}_I^\dagger(y)\big)\,|0\rangle \;=\; D_F(x-y;M^2) \;=\; \int \frac{d^4p}{(2\pi)^4}\,\frac{i\,e^{-ip\cdot(x-y)}}{p^2-M^2+i\epsilon} \qquad (\epsilon > 0 \text{ infinitesimal})$$

and the one for the ϕ field by $D_F(x-y;m^2)$.

Hint: use that $\langle 0|\, T\big(\hat{\psi}_I(x)\hat{\psi}_I(y)\big)\,|0\rangle = \langle 0|\, T\big(\hat{\psi}_I^\dagger(x)\hat{\psi}_I^\dagger(y)\big)\,|0\rangle = 0$ according to exercise 5(b).

(c) Write the answer in part (b) in terms of Feynman diagrams. Use a solid line for the ψ propagators and a dotted line for the ϕ propagators. Use the following extra drawing convention: draw an arrow on ψ propagators, representing the direction of particle-number flow. Hence, the ψ-particles are defined to flow along the arrow, whereas ψ-antiparticles flow against it!

Explain in this context why the operator $\hat{\psi}_I(x)$ corresponds to an arrow that flows into the external/internal point x, whereas $\hat{\psi}_I^\dagger(x)$ corresponds to an arrow that flows out of the external/internal point x.

(d) – Extract the Feynman rules in position space.
– Why do we not have to worry about symmetry factors?

(e) Translate these Feynman rules to the momentum representation.

Exercise 8: More on the arrow convention for particles and antiparticles

In exercises 5 and 7 you have seen various aspects of particle flow (arrows) in the scalar Yukawa theory, pertaining to internal/external points as well as propagators. Use these aspects to explain why the arrows in the associated Feynman diagrams link up to form a continuous flow.

What conservation law is actually causing this phenomenon?