Quantum Field Theory Exercises week 7

Exercise 9: dealing with matrix elements in the scalar Yukawa theory

Consider the scalar Yukawa theory, for which you have derived some Feynman rules in exercise 7.

(a) Derive the additional momentum-space Feynman rules that are needed in order to calculate amplitudes in this theory. Explain in this context why you should treat ψ-particles and $\bar{\psi}$-particles (i.e. ψ-antiparticles) separately.

Use the same drawing convention as introduced in exercise 7: draw an arrow on ψ and $\bar{\psi}$ lines, representing the direction of particle-number flow. Hence, the ψ-particles are defined to flow along the arrow, whereas $\bar{\psi}$-particles flow against it! Use solid lines to indicate $\psi/\bar{\psi}$-particles and dashed lines for the ϕ-particles.

If you have derived the additional Feynman rules correctly you should have found that $\hat{\psi}_I(x)$ corresponds to an arrow that flows into the interaction vertex, whereas $\hat{\psi}_I^\dagger(x)$ corresponds to an arrow that flows out of the interaction vertex!

(b) Use these Feynman rules to calculate the amplitude for the decay process

$$\phi(k_A) \;\to\; \psi(p_1)\bar{\psi}(p_2)$$

to <u>lowest non-vanishing order</u>, which is usually referred to as the lowest-order amplitude. To this end you may use that at lowest non-vanishing order $i\mathcal{M}$ = sum of all fully connected amputated Feynman diagrams in momentum space.

Compare to page 36 of the lecture notes and judge for yourself whether you like Feynman rules or not.

(c) Calculate the lowest-order amplitudes for the following scattering processes:

$$\psi(k_A)\psi(k_B) \;\to\; \psi(p_1)\psi(p_2)\ ,$$

$$\psi(k_A)\bar{\psi}(k_B) \;\to\; \phi(p_1)\phi(p_2)\ ,$$

$$\psi(k_A)\bar{\psi}(k_B) \;\to\; \psi(p_1)\bar{\psi}(p_2)\ ,$$

where k_A and k_B are the momenta of the initial-state particles and p_1 and p_2 the momenta of the final-state particles. Compare the matrix elements for the first and third process, and indicate how the third matrix element can be obtained directly from the first one. This way of switching from particles in the initial/final state to antiparticles in the final/initial state (and vice versa) is called <u>crossing</u>.

(d) Explain why the lowest-order amplitude vanishes for the scattering process

$$\psi(k_A)\psi(k_B) \;\to\; \phi(p_1)\phi(p_2)\ .$$

Exercise 11: decay rates, cross sections and reaction channels

Read § 2.8.3 of the lecture notes about CM kinematics and consider the scalar Yukawa theory.

(a) Calculate the lowest-order decay width $\Gamma_{\phi\to\psi\bar{\psi}}$ in the rest frame of the decaying particle. For which masses is this decay possible?

(b) Consider the process $\psi\bar{\psi} \to \phi\phi$ in the centre-of-mass frame.

- Which channels contribute to this process?
- What do the corresponding diagrams say about the θ-dependence?
- For which centre-of-mass energies can this process occur?
- Calculate the lowest-order differential cross section $\left(\mathrm{d}\sigma/\mathrm{d}\Omega\right)_{\rm CM}$.

(c) Consider the process $\psi\bar{\psi} \to \psi\bar{\psi}$ in the centre-of-mass frame.

- Which channels contribute to this process?
- What are the ranges of the Mandelstam variables?
- For which centre-of-mass energies can this process occur?
- Calculate the lowest-order differential cross section.
- Assume that $m > 2M$. Look at the energy dependence of both reaction channels separately. Do you notice something special in one of them?
- Use the other reaction channel to determine whether the Yukawa interaction between ψ-particles and $\bar{\psi}$-particles is attractive or repulsive.

  Hint: don't perform an explicit calculation, just use the analogy with the calculation that is presented on pages 55 and 56 of the lecture notes.